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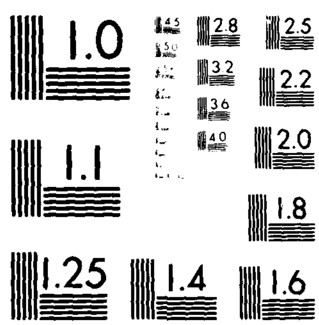

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An Analytical Solution of the Two Star
Sight Problem of Celestial Navigation

by

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February 1981

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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER U. of Iowa 81-2	2. GOVT ACCESSION NO. AD-A098626	3. RECIPIENT'S CATALOG NUMBER --
4. TITLE (and Subtitle) An Analytical Solution of the Two Star Sight Problem of Celestial Navigation		5. TYPE OF REPORT & PERIOD COVERED Progress, February 1981 6. PERFORMING ORG. REPORT NUMBER --
7. AUTHOR(s) JAMES A. VAN ALLEN		8. CONTRACT OR GRANT NUMBER(s) N00014-76-C-0016
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Physics and Astronomy The University of Iowa Iowa City, Iowa 52242		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS --
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Electronics Program Office Arlington, Virginia 22217		12. REPORT DATE February 1981 13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

To be published in Navigation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Celestial Navigation
Two Star Sight Problem

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

[See page following.]

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ABSTRACT

The classical problem of celestial navigation, in its simplest form, is the determination of an observer's longitude and latitude from the altitudes of two identified stars, observed at a known Greenwich Mean Time on a known date. A novel solution of this problem in closed analytical form is given herein. The solution yields the two possible positions of the observer without any prior knowledge of his position, without any dependence on tables of computed altitudes and azimuths for an assumed position, and without any graphical work. The basic two-fold ambiguity is resolved to yield a single, unique position by repeating the calculation using the altitudes of a third star and of one of the two previous stars. The full analytical solution is given as are some artificial numerical examples, readily performed on a hand-held, programmable calculator.

p 3

1. INTRODUCTION AND SUMMARY

The classical problem of celestial navigation, in its simplest form, is the determination of an observer's longitude and latitude from the altitudes of two identified stars, observed at a known Greenwich Mean Time on a known date. Numerous ingenious solutions to this problem by essentially trial-and-error methods using the concept of the "assumed position" have been devised and reduced to practice (Bowditch, 1962). But, to the author's knowledge, no treatise on practical navigation draws attention to the fact that the basic problem can be solved in closed analytical form without reference to tables of computed altitudes and azimuths, graphical approximations, etc. At least two analytical solutions to this problem have been published (Kotlaric, 1971-72) (Watkins and Janiczek, 1978-79). The solution given herein appears to have merit because of its geometric simplicity, its freedom from trigonometric ambiguities, and its ready adaptability to a hand-held, programmable calculator.

Data from one pair of star sights yield two possible positions, usually widely separated and easily distinguished. Data from a second pair of star sights (i.e., using the altitudes a third star and of one of the two previous stars) resolve the basic ambiguity because only one of the two positions from the second calculation is identical to one of the two positions from the first calculation. No prior knowledge of the approximate position of the observer is required.

2. STATEMENT OF THE PROBLEM

Observed Data

h_1, h_2	Altitudes (corrected for atmospheric refraction, etc.) of stars 1, 2
t	Greenwich Mean Time of observation (on a known date)

For simplicity, it is assumed that the two observations are simultaneous.

Data Derived from Observations and from the AIR ALMANAC

λ_1, λ_2	East longitudes of substellar points of stars 1, 2
δ_1, δ_2	Declinations of stars 1, 2
ζ_1, ζ_2	Zenith distances of stars 1, 2

Problem

To find the Longitudes, Λ (East + or West -), and Latitudes, Φ (North + or South -) of both intersections (P and Q) of the two small circles of position on a sphere of unit radius (Figure 1).

3. AN ANALYTIC SOLUTION

(a) Each circle of position is the intersection of the circular cone of half angle ζ with the unit sphere (Figure 2). The center of the circle has the spherical coordinates of the star λ, δ (Figure 3). The equation of the plane containing the circle of position is of the form

$$x \cos\alpha + y \cos\beta + z \cos\gamma - p = 0 \qquad (1)$$

where α, β, and γ are the angles to the X, Y, and Z axes, respectively, of the normal to the plane, this normal being through the origin of coordinates O, and p being the distance along that normal from O to the plane (Figures 2 and 4). The adopted XYZ coordinate system is a right-handed one with O at the center of the earth, with Z the earth's polar axis (+Z to the North Pole), with the XY plane the equatorial plane, and with +X through the Greenwich meridian. Note that

$$p = \cos\zeta$$

and that

$$\gamma = 90 - \delta .$$

Thus

$$\cos\alpha = \cos\lambda\cos\delta$$
$$\cos\beta = \sin\lambda\cos\delta$$
$$\cos\gamma = \sin\delta$$

After substituting the above quantities in equation (1), the equation of the plane is found to be:

$$x(\cos\lambda_i\cos\delta_i) + y(\sin\lambda_i\cos\delta_i) + z\sin\delta_i - \cos\zeta_i = 0, \quad i = 1, 2\,. \tag{2}$$

(b) The two planes intersect in the line PQ.

Let $a_i \equiv \cos\lambda_i\cos\delta_i$

$b_i \equiv \sin\lambda_i\cos\delta_i$

$c_i \equiv \sin\delta_i$

$p_i \equiv \cos\zeta_i$

The equation of this line of intersection is obtained by the simultaneous solution of equations (2) for i = 1 and i = 2 to give:

$$x = \frac{-Bz + C}{A} = \frac{-Ey + F}{D} \tag{3}$$

wherein

$$A \equiv (a_1b_2 - a_2b_1)$$
$$B \equiv (b_2c_1 - b_1c_2)$$
$$C \equiv (b_2p_1 - b_1p_2)$$
$$D \equiv (a_1c_2 - a_2c_1)$$
$$E \equiv (b_1c_2 - b_2c_1)$$
$$F \equiv (c_2p_1 - c_1p_2)$$

Equations (3) may be regarded as parametric equations of the line PQ with x the parameter that specifies the position of a point on that line. The other two coordinates of that point are given by:

$$y = \frac{F - Dx}{E}$$

$$z = \frac{C - Ax}{B} \qquad (4)$$

(c) The two possible positions of the observer, P and Q, are the intersections of the above line with the unit sphere; that is, they are the two points for which

$$x^2 + y^2 + z^2 = 1 \qquad (5)$$

(d) Substitutions from Equations (4) into Equation (5) yield a quadratic equation in x, which has two real roots, x_P and x_Q.

(e) Equations (4) are then used to find the other two coordinates of P and Q:

$$P : (x_P, y_P, z_P)$$

$$Q : (x_Q, y_Q, z_Q)$$

(f) Finally, the rectangular coordinates of P and Q are converted to spherical coordinates to yield the desired longitude and latitude of each point, viz.

$$P : \Lambda_P, \Phi_P$$

$$Q : \Lambda_Q, \Phi_Q$$

Q.E.D.

4. ARTIFICIAL EXAMPLES

0000 GMT of 1 September 1975

(a) Input Data for $\Lambda = -91^\circ.532$, $\Phi = +41^\circ.662$

Arcturus

$\lambda = -125^\circ.915$
$\delta = +19.317$
$\zeta = 36.704$

Altair

$\lambda = -42^\circ.156$
$\delta = +8.799$
$\zeta = 54.382$

Antares

$\lambda = -92^\circ.581$
$\delta = -26.376$
$\zeta = 68.045$

Vega

$\lambda = -60^\circ.520$
$\delta = +38.759$
$\zeta = 23.731$

(b) Calculated Positions from Pairs of Star Sights

Arcturus-Altair

$\Lambda_P = -91^\circ.532$
$\Phi_P = +41.661$

$\Lambda_Q = -95^\circ.605$
$\Phi_Q = -2.148$

Arcturus-Antares

$\Lambda_P = -91^\circ.532$
$\Phi_P = +41.662$

$\Lambda_Q = -157^\circ.841$
$\Phi_P = +0.136$

<u>Arcturus-Vega</u>

$$\Lambda_P = -\ 91^{\circ}.532$$
$$\Phi_P = +\ 41.661$$

$$\Lambda_Q = -\ 86^{\circ}.950$$
$$\Phi_P = +\ 29.334$$

<u>Vega-Antares</u>

$$\Lambda_P = -\ 91^{\circ}.532$$
$$\Phi_P = +\ 41.662$$

$$\Lambda_Q = -\ 42^{\circ}.186$$
$$\Phi_Q = +\ 21.009$$

<u>Vega-Altair</u>

$$\Lambda_P = -\ 91^{\circ}.532$$
$$\Phi_P = +\ 41.662$$

$$\Lambda_Q = -\ 55^{\circ}.550$$
$$\Phi_Q = +\ 62.295$$

<u>Altair-Antares</u>

$$\Lambda_P = -\ 91^{\circ}.532$$
$$\Phi_P = +\ 41.662$$

$$\Lambda_Q = -\ 11^{\circ}.087$$
$$\Phi_Q = -\ 37.143$$

(c) <u>Remarks</u>

It is clear that one of the two calculated positions in each of the six cases is identical to the position that was used in the construction of these examples. This is the only solution that is common to two or more cases.

The method is, of course, applicable to observations of the sun, moon, and planets as well as to observations of stars.

4. ACKNOWLEDGEMENT

The author thanks Mark M. Baumback for helpful discussion and the U. S. Office of Naval Research for support.

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R. Watkins and P. M. Janiczek, Sight reduction with matrices, Navigation, Vol. 25, 447-448, Winter 1978-79.

CAPTIONS FOR FIGURES

Figure 1. This is a sketch of two circles of position on a unit sphere whose center is the center of the earth. The centers 1 and 2 of the two circles of position have the spherical coordinates λ_1, δ_1 and λ_2, δ_2 of the substellar points of stars 1 and 2, respectively. The line PQ is the intersection of the two planes containing the respective circles of position. The intersections of this line with the unit sphere yield the two possible positions of the observer, P and Q.

Figure 2. This diagram shows a cross-section of the unit sphere that contains the origin O and the substellar point S of an observed star. The circle of position is the intersection with the sphere of the circular cone whose axis is OS and whose half angle is ζ. The normal to this plane OR has length p.

Figure 3. The east longitude λ and declination δ (or latitude) of the substellar point are shown relative to the coordinate system in which +Z is toward the north celestial pole, the XY plane is the earth's equatorial plane, and the +X axis lies in the Greenwich meridian.

Figure 4. In the same coordinate system as that in Figure 3, α, β, and γ are the angles the line from O to the star makes with the X, Y, and Z axes, respectively.

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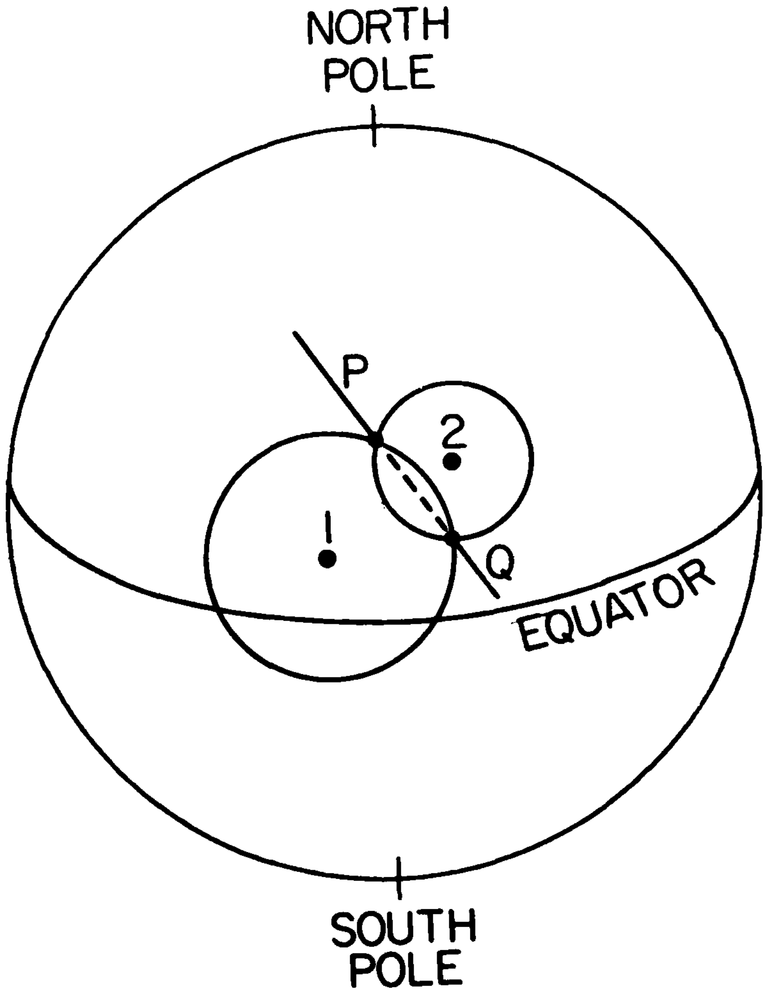

Figure 1

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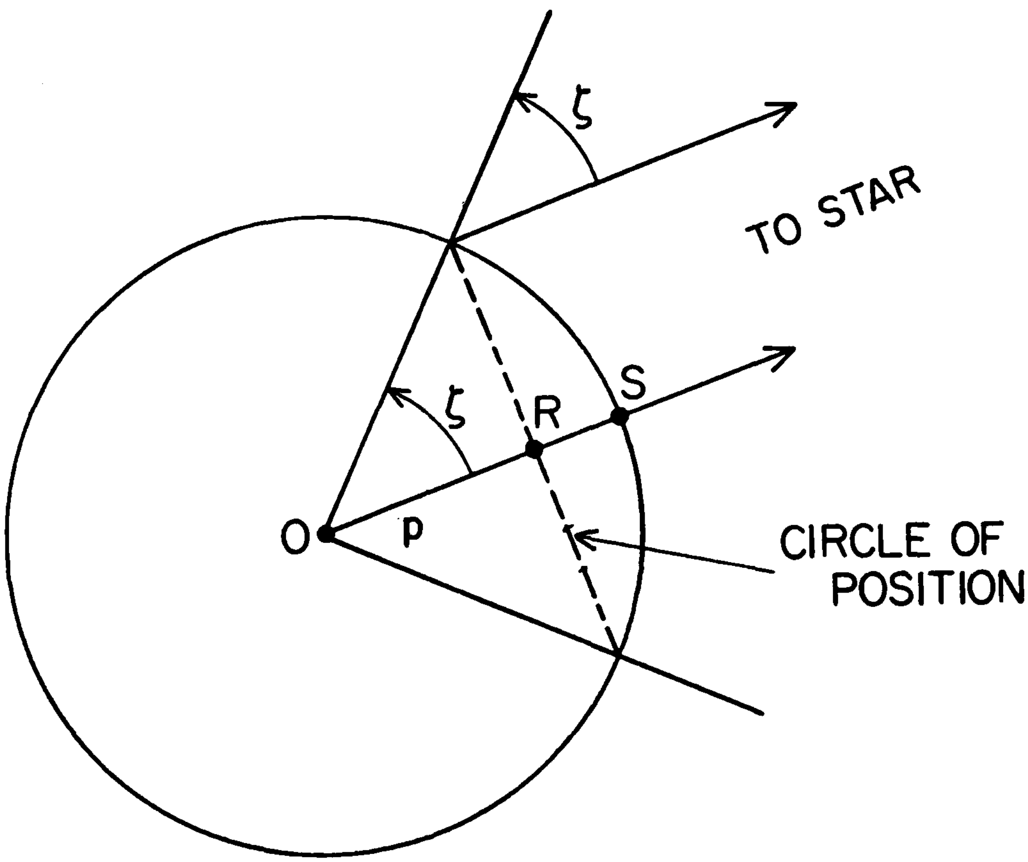

Figure 2

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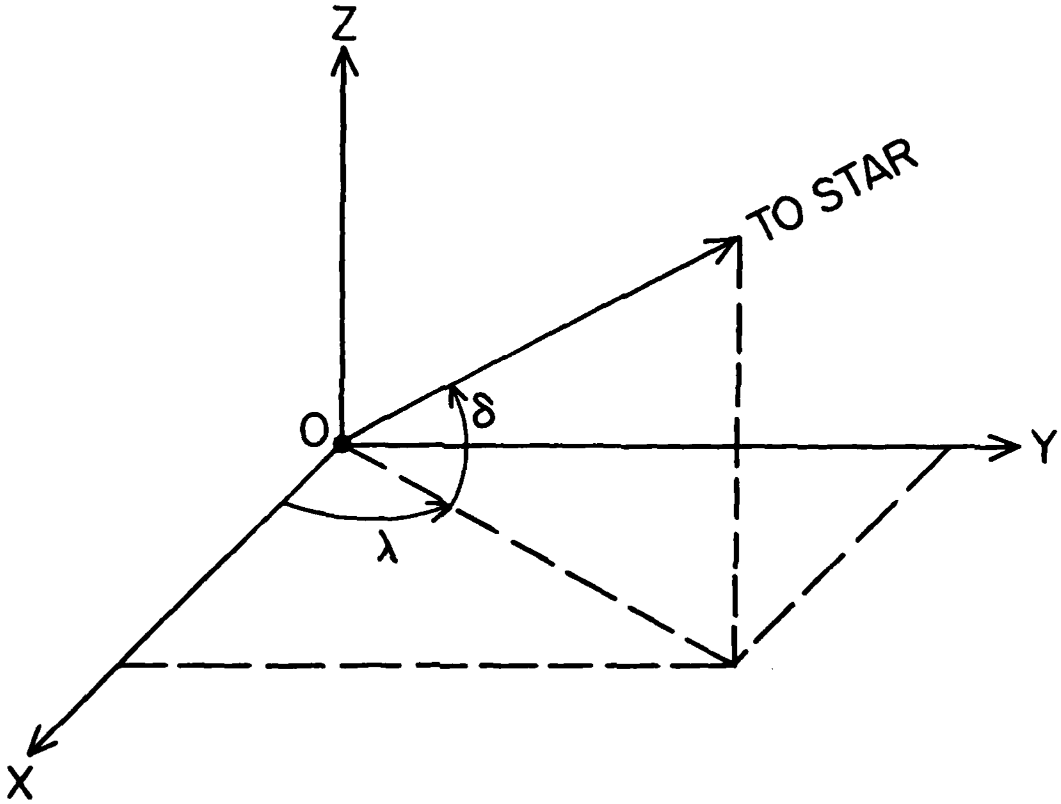

Figure 3

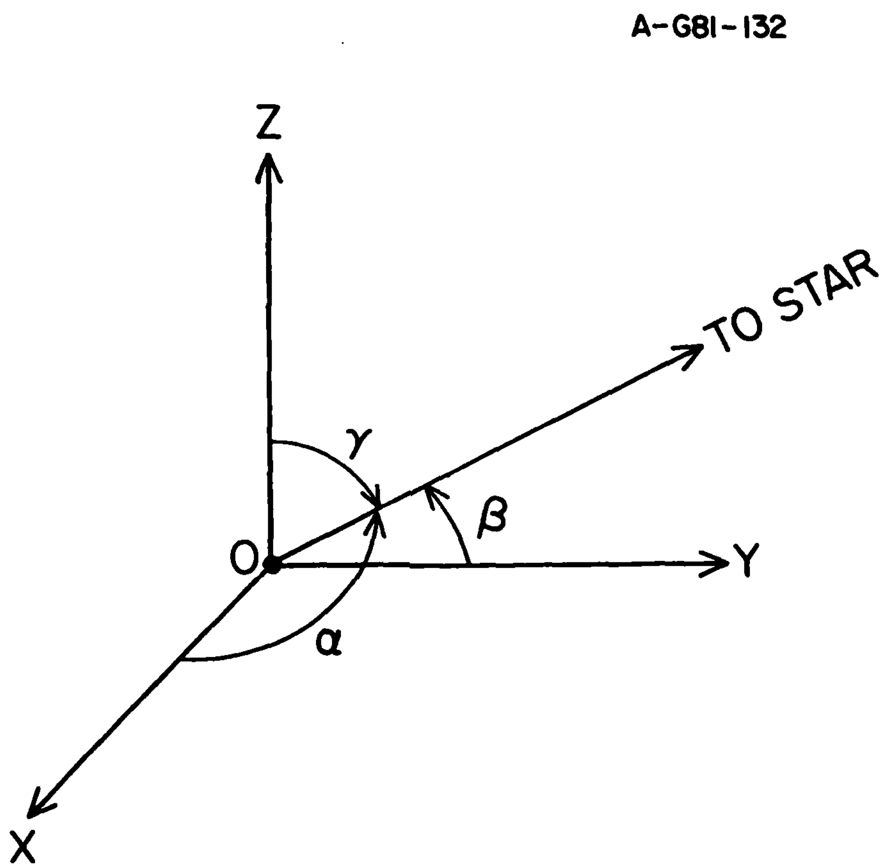

Figure 4